ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

It is essential that candidates check that they have answered each question fully and given their final answer in the form required. Too many candidates still do not appreciate the meaning of giving an answer in exact form, choosing instead to give their answer correct to 3 significant figures. This is the accuracy that is required, as per the rubric, when an exact answer, or given form, is not required. It is also essential that where the use of a calculator is not permitted, that candidates show sufficient working to justify non-use. Candidates should also be guided by the mark allocation for a given question part as this will give them an idea of the amount of work expected to obtain a solution.

General comments

Most candidates attempted all the questions. There did not appear to be any timing issues. Some candidates needed extra space for either the continuation of a solution, or to make another attempt at a solution. It was pleasing to see that these candidates worked these solutions on additional sheets, enabling clear working to be shown.

Comments on specific questions

Question 1

(a) Most candidates gained marks for the coordinates of the intercepts with the coordinate axes. Although most recognised the correct basic shape of the cubic function, many placed the minimum stationary point on the *y*-axis, rather than in the fourth quadrant.

(b) Correct values of *x* were obtained by most candidates, making use of their graph from **part (a)**.

Question 2

Many completely correct solutions were seen. The most common method and by far the simplest method was to equate the two given equations, form a quadratic equation equated to zero and then make use of the discriminant of this quadratic equation. Other methods included equating gradients and substitution of the resulting relationship into an appropriate equation to either the value of *k*, or the value of *x* at the points where the tangent met the curve and hence *k*.

Some candidates, having found the critical values for k, went on to form an inequality, showing a basic misunderstanding of the problem.

Question 3

This was well done by most candidates, with many obtaining full marks. Any errors were usually due to arithmetic slips or an error in the calculation of the constant c. This latter error was usually due to the calculation of ${}^5C_2 \times 3^3 \times (-a)$ or ${}^5C_2 \times 3^3 \times a$ rather than the correct ${}^5C_2 \times 3^3 \times (-a)^2$.

Question 4

Too many candidates were unable to gain full marks in this question despite using a completely correct method throughout. This was because their working throughout their solution was not exact, the value of the *y*-coordinate at the point on the curve where $x = 2$ and subsequent work being given in decimal form. It is suggested that centres ensure that their candidates understand and appreciate the meaning of the word 'exact' in the context of a mathematics examination and that their candidates should check each question carefully to ensure that they have given final answer in the correct form.

Very few errors in the differentiation needed to calculate the gradient at the point $x = 2$, were seen, with most candidates obtaining a correct gradient and then continuing on the fine the equation of the tangent.

Question 5

Some candidates did not show sufficient work to justify the non-use of a calculator.

(a) Most candidates were able to find the area of the triangle as requested. To justify the non-use of a calculator, it was expected that $\frac{1}{2}\left(5-\sqrt{3}\right)\left(2+4\sqrt{3}\right)$ be expanded to show at least $\frac{1}{2}\left(10-18\sqrt{3}-12\right)$ or $\left(5-9\sqrt{3}-6\right)$. Many candidates chose to show all four terms in their expansions which was even better.

(b) Correct expressions for $\tan ABC$ were common. Again, it was necessary to see sufficient evidence of non-calculator use with a minimum of $\left(5-11\sqrt{3}+6\right)$ being seen in the numerator. There were some errors with candidates using the length of *BC* rather than the length of *BD*.

(c) A mark allocation of two marks was intended to inform candidates that very little work was necessary. Many recognised that they needed to make use of the trigonometric identity $\sec^2 ABC = \tan^2 ABC + 1$, which then yielded the required result in a couple of lines. Some candidates chose to use $\sec^2 ABC = \left(\frac{\sqrt{\left(5-\sqrt{3}\right)^2+\left(1+2\sqrt{3}\right)^2}}{1+2\sqrt{3}}\right)^2$. Full credit was given for this method provided sufficient detail in the necessary expansions and rationalisation was seen. It was a considerably longer method, which should have alerted candidates to the fact that there may be a much quicker method.

Question 6

(a) Very few errors were seen in this part of the question, with candidates recognising the necessary steps needed to obtain the equation of the perpendicular bisector. Some candidates were unable to gain the final accuracy mark as they did not check that they had given their final answer in the required form of $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(b) Most candidates obtained the available mark for substitution of $x = 5$ into their perpendicular bisector equation to obtain the value of *p*.

(c) It was intended that candidates made use of a displacement vector or similar method involving *x* and *y* distances to find the coordinates of the point *D*. Again, a mark allocation of two marks should have alerted candidates to a straightforward method being available. Some candidates made use of a displacement vector but ended up with an incomplete method, finding the coordinate of the mid-point of *AB*. In questions of this type, if a straightforward method is not recognised, it is suggested that a small sketch be drawn of the situation, with points and coordinates marked in to clarify where the point D is actually located.

Question 7

(a) Most candidates made the appropriate substitutions into the given function and solved the resulting equations correctly to give the correct value of *a* and of *b*. Any errors were usually arithmetic. The candidates were told that *a* and *b* were integers so if non-integer answers are obtained, a careful check of working to identify possible errors should be made.

(b) A correct answer of $(2x+1)(3x^2-12)$ was obtained by most candidates, either by the use of algebraic long division or by the use of observation. Some candidates chose to use synthetic division, with many of them not taking the factor of 2 into account, thus ending up with the result $(2x+1)(6x^2-24)$. A check of the coefficient of x^3 should alert candidates to a possible factor error.

(c) Many correct solutions were obtained, with the occasional omission of the root $x=\frac{1}{2}$.

Question 8

(a) Too many candidates do not recognise that fact a vector is equal to a magnitude multiplied by a unit direction vector. The incorrect answer $\begin{pmatrix}-260\\624\end{pmatrix}$ was all too common.

(b) Most candidates realised that the position vector of *P* was a product of their answer to **part (a)** and *t*.

(c) A correct method was used by most to obtain the correct position vector of *Q* at time *t*.

(d) A subtraction of the answer to **part (a)** from the answer to **part (c)** was attempted by most. The difference in velocity vectors of *P* of $\begin{pmatrix}-260\\624\end{pmatrix}$ and *Q* should have alerted candidates to a possible error.

(e) Most candidates attempted to find the modulus of their answer to **part (d)**. Not obtaining the given result should mean that previous work needs to be checked, rather than contriving to obtain the given result.

(f) This part of the question was an easy two marks, especially if candidates had been unable to obtain the given result. Unfortunately, some candidates did not attempt it, having been unable to obtain the correct result in **part (e)**. Candidates should always make use of given results in following parts even if they are unable to obtain the given result. It was necessary to identify the correct value of *t* from the quadratic equation rather than give both solutions.

Question 9

(a) (i) Nearly all candidates obtained a correct solution.

(ii) Nearly all candidates obtained a correct solution.

(iii) In a question of this type, it is essential that the method used is stated clearly to enable examiners to give credit where necessary. Just writing down sets of figures with no explanation is meaningless unless these figures can be identified easily. Simple statements such as '*Starts with 8 and ends with an odd number*' followed by the appropriate working would aid the marking process. Many candidates did however, show sufficient detail and were able to gain some credit and, in many cases, full marks.

(b) Many completely correct solutions were seen with correct manipulation of the factorial terms and solution of the resulting quadratic or cubic equation.

Question 10

(a) Most candidates were able to obtain two correct solutions to the required level of accuracy. Any errors usually involved the use of incorrect quadrants in finding the solutions.

(b) (i) Most candidates were able to deal with the terms on the left-hand side of the given expression correctly, by forming a single fraction with a denominator of $\sin^2\theta - 1$. Unfortunately some candidates then wrote the denominator of the fraction as $\cos^2\theta$ rather than $-\cos^2\theta$, thus obtaining an incorrect answer of $2\sec^2\theta$ rather than the correct answer of $-2\sec^2\theta$.

(ii) For those candidates with a correct result from **part (i)**, many were able to obtain at least two correct solutions, provided they had recognised that they were ultimately solving the equations $\cos 3\phi = \pm\frac{1}{2}$. Solution of the equation $\cos 3\phi = \frac{1}{2}$ usually yielded two correct solutions and gained 3 marks.

Those candidates who obtained the result of $2\sec^2\theta$ from **part (i)**, should have realised that an error had been made when an equation of $2\sec^2 3\phi = -8$ or similar was obtained. This equation has no solutions and it was essential that candidates recognise this and check previous work for errors. Most of these candidates unfortunately did not do this, merely deleting the negative sign. This resulted in an incorrect solution.

Question 11

This question tested the new syllabus content on integration. It was pleasing to see many fully correct solutions, with correct integration, application of limits and logarithmic manipulation to obtain the result $\ln\left(\frac{(2a+3)(3a-1)}{10a}\right) = \ln 2.4$ or equivalent. Errors usually involved incorrect signs or incorrect coefficients, although there were a few candidates who did not recognise that logarithms were involved in the integration. It was also pleasing to see that the possible root of $-\frac{1}{6}$ was rejected by most.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

In order to succeed in this examination, candidates need to read each question carefully and ensure that they have identified key statements and information. Candidates also need to show sufficient method so that marks can be awarded. Attention should be given to the instructions on the front page of the examination paper. Candidates who do not show full method because they have used their calculator to perform key operations, such as finding the value of the derivative of a function for a particular value, will not be credited. Candidates should ensure that their answers are given to at least the accuracy required in a question. When no particular accuracy is asked for, candidates should ensure that they follow the instructions on the front page. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions.

General comments

Most candidates were very well prepared for this examination. Candidates were able to recall and apply techniques in order to solve problems when needed. Some questions were unstructured and needed several techniques to be applied in order to solve them fully. Many candidates were able to do this successfully.

Many candidates attempted complete solutions, with sufficient working shown to gain full credit. Occasionally, candidates rounded working values to three significant figures. This resulted in a premature approximation error and a loss of final accuracy. In order for final answers to be accurate to three significant figures, working values must be given to a greater accuracy. This was evident in **Question 6(b)**, **Question 9** and **Question 11** in this paper.

When candidates are required to "Explain why" a statement is correct, it is important that any explanation is not contradictory or does not contain incorrect statements. This was especially the case in **Question 8(a)** in this paper.

Notation used should be correct and expressions unambiguously stated. For example, the domain of a function with argument *x* should be stated in terms of *x*. Use of brackets or correct ordering of terms in a product should be used to ensure that terms cannot be misinterpreted. This was required in **Question 9** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used the blank page at the end of the examination paper or used additional paper. This was very sensible as their work remained well presented and could be marked without difficulty. Candidates who did this usually added a comment in the answer space in their main script to indicate that their answer was written, or continued, elsewhere. This was very helpful.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

The majority of candidates were able to expand and simplify the terms of the inequality and use the quadratic expression to find the correct critical values. Most were able to express this as a single inequality. The few candidates who gave the answer as two separate inequalities which were connected with 'and' earned full credit. Otherwise, candidates who gave two separate inequalities connected by 'or' or a comma did not earn the final accuracy mark.

Question 2

Many excellent and fully correct solutions were seen to this question. A few candidates made slips in their method, often when finding the y-intercept of the straight line. A few other candidates misinterpreted lg$y$ as ln$y$ and gave their final answer as a power of e. Some candidates would have benefitted from reading the question more carefully as they found lg$y$ as a function of x and so did not complete the solution. Weaker candidates tended to use lg(7) and 6^3, for example, in their calculations of the gradient and intercept of the straight line. These candidates may have been helped if they had made a simple sketch to interpret the information given. This was rarely seen.

Question 3

Candidates needed to observe that the equation given was quadratic in 3^x. Many candidates did see this and produced a clear and accurate solution. An exact answer was required and if this was not seen, full credit was not earned. A few candidates generally either attempted to take logarithms of each term in the equation or attempted to combine the powers of 3 in a spurious way.

Question 4

Candidates who used the vector relationship $\overrightarrow{OC} - \overrightarrow{OA} = 4(\overrightarrow{OC} - \overrightarrow{OB})$ were usually successful in finding the vector $\overrightarrow{OC}$. Many of these candidates went on to find the magnitude of $\overrightarrow{OC}$ and complete the solution correctly. A small number of candidates incorrectly multiplied $\overrightarrow{OC}$ by its magnitude, rather than dividing. A few candidates needed to reread the question, or had possibly misinterpreted the question, as they made no attempt to continue their solution after they had found $\overrightarrow{OC}$. Some candidates attempted to use other vector routes or equations such as $\overrightarrow{OC} = \overrightarrow{OA} + \frac{4}{3}(\overrightarrow{OB} - \overrightarrow{OA})$ or $\overrightarrow{OC} = \overrightarrow{OB} + \frac{1}{3}(\overrightarrow{OB} - \overrightarrow{OA})$. These were less successful as, for example, they often found $\overrightarrow{AB}$ but then multiplied this by an incorrect scalar, misinterpreting the information given. These candidates may have found it helpful to have made a simple sketch.

Question 5

(a) Almost all candidates drew a ruled graph with correctly positioned vertex and y-intercept. Many candidates also labelled the coordinates of the points of intersection with the axes as required. Those candidates who omitted to do this should be aware that, for example, a vertex positioned between 1 and 2 is insufficient to imply that the vertex is at 1.4. The question required that the y-intercept and the vertex should have been clearly indicated. A few candidates positioned the vertex on the negative x-axis or had the y-intercept labelled as 2. Candidates who did not use a straight edge to draw the two linear sections of the graph did not earn full credit as their attempts were not sufficiently linear or pointed at the vertex to earn the first mark. Only very few candidates attempted to draw a curve rather than two linear sections.

(b) A very high proportion of candidates applied a correct order of operations to form a correct pair of linear equations or a correct quadratic equation to solve. Most of these candidates were able to solve their equations or equation and very few errors were seen. On occasion, candidates incorrectly negated 14 and then rearranged, giving the answer 0.88.

Question 6

(a) A high proportion of candidates formed a correct equation for the perimeter and solved it to find θ.

A few candidates equated the arc length to the perimeter and would possibly have benefitted from making a sketch. Other candidates incorrectly expanded the brackets for $2(6 + 5\pi)$. These candidates need to understand that it is important to write an unsimplified form down first before attempting to simplify. Candidates who converted to decimal form occasionally made rounding errors, whereas those who kept their working in terms of π did not.

(b) Many good and accurate solutions were seen. Commonly candidates either formed a right-angled triangle or used the cosine rule to find the length of the chord *AB*. On occasion, when using the right-angled triangle approach, candidates doubled $\frac{\pi}{4}$ instead of halving it. The arc length was almost always stated correctly. A few candidates made premature approximation errors in their working values and gave an answer outside the acceptable range. A very small number of candidates needed to read the question more carefully as they found the area of the shaded region, rather than the perimeter, as was required.

Question 7

This question was very well answered and very few slips were seen. Many candidates showed very little method, particularly when solving their equation. This was not penalised on this occasion. However, candidates should be advised that showing little method, especially when an error has been made, is unlikely to result in full marks being awarded. It is also very evident that some candidates are working back from answers obtained using a calculator. This also is not recommended as a method of solution. Calculators are valuable checking tools when solving equations, but should not replace the understanding needed to solve the given problem, which should be shown.

Question 8

(a) The simplest explanation was to state that each value of *x* had a unique value of *y*. A good number of candidates did exactly that. A few candidates stated that the graph showed a many to one mapping, which was condoned; stating that f passed the vertical line test was also condoned. Incorrect statements such as ‘f is a one-one function’ were not accepted. Some candidates contradicted themselves, making a correct statement and also an incorrect statement such as ‘f is one to many’. Some candidates considered that f was a function as it had input and output or a domain and range, misunderstanding what was required.

(b) Candidates simply needed to read the values from the graph and compose the correct inequality. A very good proportion of candidates were able to do this. Errors, when seen, were: the inequality being given in *x* not f, the range being stated as two inequalities that were not joined using ‘and’ or the inequality signs being incorrect in some way.

(c) The majority of candidates were able to state the values of *a* and *c* correctly. A good number of candidates were also able to find the correct value of *b*. A few candidates found the value of *b* to be more challenging, however, and it was not uncommon to see, for example, $2\pi b = \frac{8\pi}{3}$. Some candidates gave *c* as 1, possibly thinking it to be the value of the *y*-intercept rather than the position of the midline or principal axis. A few candidates completely misinterpreted each of *a*, *b* and *c*, often interchanging them. Testing values may have helped these candidates.

Question 9

This question involved the application of several rules of differentiation. Commonly candidates used the chain, product and quotient rules. A very good number of candidates earned four or more of the six marks available in this way. Many were able to earn the first three marks without issue. Fewer candidates earned the fourth mark, which required an unambiguous form of the derivative. This was most easily achieved by writing the trigonometric term in each product as the final term, for example, $\frac{x^2\left(3e^{3x}\sin x + e^{3x}\cos x\right) - 2x(e^{3x}\sin x)}{x^4}$ or by showing a correct calculation such as $\frac{2.5947 - 2.1486}{0.0625}$.
Candidates who chose to rewrite the given function and apply the product rule twice, or once to the triple product, were less successful as there were more sign errors. Some candidates were using their calculator in degree mode when it was essential that they were in radian mode. It is advised that, before starting any question where an angular measure is used, candidates check that their calculator is in the appropriate mode. On occasion, a ‘correct’ value for $\frac{dy}{dx}$ when $x = 0.5$ appeared following incorrect differentiation. Clearly this value had been found using the numerical differentiation function on a calculator and this was not allowed full credit. Full marks were only awarded to candidates who had given clear evidence that they were able to differentiate correctly and then complete the solution.

Question 10

(a) (i) This part was almost universally correctly answered. Occasional slips in selection of operation were seen, but these were rare.

(ii) Again, this was very well answered. Occasionally candidates used incorrect notation and gave an inequality in x which was not accepted as a range had been requested. A few candidates embellished their otherwise correct answer with an incorrect statement such as $y \neq 3$. This was not accepted.

(iii) Many candidates found this part of the question to be challenging. An inequality in terms of x, as a domain was required, should have been given. Many candidates stated that the domain of g^{-1} was the range of g. However, few candidates were able to deduce that x had to be greater than 3 and less than or equal to 4. Some candidates were able to offer either $x > 3$ or $x \leqslant 4$. The most commonly seen incorrect answers were $x \neq 3$ or $x \geqslant 4$.

(b) A good number of candidates earned three or four marks in this part of the question. Many candidates were able to draw an acceptable graph for h and reflect this in the line $y = x$ to produce a graph for h^{-1}. It was important that the graph of the inverse function was a genuine attempt at a reflection and so clear errors in intercepts or length of curves were not condoned. A good number of candidates drew graphs that started on the x and y axis for h and h^{-1} respectively. However, not all candidates indicated the intercepts as $\frac{2}{3}$ on each axis. Candidates who wrote $\frac{2}{3}$ as a decimal and rounded were penalised, for example 0.7 was not accepted. Some candidates drew sections of each graph outside the required domain and these candidates may have improved if they had reread the question and observed the given domain for h and thus for h^{-1}. It was only possible to earn the final mark if the first two marks had been awarded, so that the graphs drawn should intersect with $y = x$ twice, as stated in the question, and be reasonable attempts at correct graphs.

Question 11

A good number of candidates earned at least six of the eight marks available for this question. The overwhelming majority of candidates used the information given about the volume of the cylinder to write h as an expression in r. A good number of these candidates formed a correct expression for the surface area and went on to differentiate and solve correctly. Some candidates included both ends of the cylinder in the surface area and may have done better if they had reread the question which stated that the cylinder was open at one end. A few other candidates used only the curved surface area and made no attempt to add either end of the cylinder. Some candidates were unable to recall the correct expression for the curved surface area. Commonly πrh was used by these candidates. Other candidates formed expressions for the surface area that were not dimensionally correct, such as $2\pi rh + 2\pi r$, which was not condoned. Many candidates were able to differentiate their expression for the surface area correctly. A few candidates arrived at a negative value for r^3. This was not reasonable since r was a dimension. Some candidates demonstrated that the value of r they had found gave the minimum value for the surface area by differentiating for a second time. This was not necessary as this was given in the question. These candidates often omitted to carry on and find the minimum value of the surface area; whilst a few others thought that the value of the second derivative, at their value of r, was the minimum value required. A few candidates made rounding errors in their working values and this resulted in their final answer being inaccurate.

Question 12

Whilst it was possible to answer this question using *suvat* equations, knowledge of such is not in the Additional mathematics syllabus and it was expected that this question would be answered using calculus.

(a) A very good number of candidates earned full marks for this part of the question. A few candidates misinterpreted a as, for example, $t - 6$ rather than -6 as given in the question. Candidates not using calculus often made sign errors.

(b) A good number of candidates were fully correct in this part. Some candidates integrated correctly but then used the limits 4 and 3 rather than 3 and 2. A simply schematic diagram may have helped to prevent this misinterpretation. Many candidates found the distance when $t = 3$ only. Again, this was a misinterpretation of the question.

Question 13

(a) (i) A reasonable number of candidates earned full credit for this part. Most candidates indicated that $ar^2 = 9$. A few of these then went on to incorrectly state $ar = 3$, which sometimes resulted in a simplification of method which was not condoned. Those candidates who wrote the sum of the first two terms as $a + ar = 10$, rather than using the sum to 2 terms formula, were much more successful. Candidates who used the sum to 2 terms formula and formed a cubic equation often included the solution $r = 1$ or $a = 9$. This was not accepted for the final accuracy marks. A few candidates formed the sum to 2 terms and the sum to 3 terms. This was a reasonable method if $a + ar + ar^2 = 19$ was used, but much more complex than necessary if the formula for the sum to 3 terms was used. The few candidates finding a first sometimes made a sign error when finding the negative value of r, giving this as positive. A few candidates found the correct two values for r but then discarded one of them. Often the reason stated was that r could not be negative or r could not be greater than 1. These candidates seem to have been thinking ahead to the next part of the question and misinterpreting $|r| < 1$.

(ii) This part of the question was reasonably well answered. Some candidates stated a 'sum to infinity' for both values of r. Other candidates gave a rounded, not an exact, answer. Candidates need to understand that, when the answer is a terminating decimal and not of great length it should be quoted in full. At the very least, candidates should quote the exact answer first, before they round, as sight of the exact answer is often enough to be credited.

(b) Almost all candidates were able to deduce that the first term was -10 and the common difference was 8, although a few candidates did make arithmetic errors. The most common approach was to then find a difference of sums. Most candidates found the sum to 200 terms of the progression correctly. Some of these candidates then found the sum to 99 terms and subtracted, or subtracted the sum to 100 terms and then added u_{100}. Many candidates, however, subtracted the sum to 100 terms only, giving 118 600 as the most common wrong answer. Some candidates found $u_{100} = 782$ and correctly used this as the first term in a sequence of 101 terms in a sum calculation. Many of the candidates using this approach, however, used 782 as the first term in a sequence of 100 terms. Similarly, those candidates who found $u_{100} = 782$ and $u_{200} = 1582$; some used 101 in their sum calculation but many used 100.